

Exercise 62

If you invest x dollars at 4% interest compounded annually, then the amount $A(x)$ of the investment after one year is $A(x) = 1.04x$. Find $A \circ A$, $A \circ A \circ A$, and $A \circ A \circ A \circ A$. What do these compositions represent? Find a formula for the composition of n copies of A .

Solution

Let $A(x) = 1.04x$. Then

$$A \circ A = A(A(x)) = 1.04(1.04x) = 1.04^2x = 1.0816x$$

$$A \circ A \circ A = A(A(A(x))) = 1.04[1.04(1.04x)] = 1.04^3x \approx 1.12486x$$

$$A \circ A \circ A \circ A = A(A(A(A(x)))) = 1.04\{1.04[1.04(1.04x)]\} = 1.04^4x \approx 1.16986x$$

$$\underbrace{A \circ \cdots \circ A}_{n \text{ copies of } A} = 1.04^n x.$$

These compositions represent the amount of money after 2, 3, 4, $\dots$, n years, respectively.